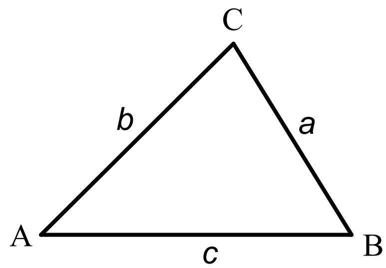
Proof of the Law of Sines

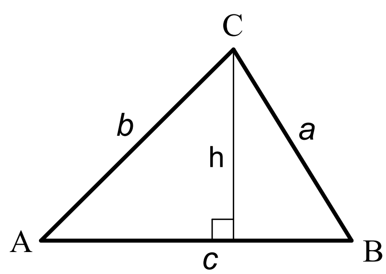
Take a triangle. Any triangle. Give the sides and angles **names**, so we can talk about them. Angles are named with an uppercase letter, because the vertex is a *point*; sides are named with lowercase letters, because they are (parts of) *lines*.

Traditionally, math people use the SAME LETTER to name an angle and the side **opposite** that angle – but the angle is the uppercase letter, and the (opposite) side is the lowercase letter. Like this:



Notice that side ***a*** is opposite angle ***A***; side ***b*** is opposite angle ***B***; etc.

Now we have a triangle; we can draw in an ***altitude*** from one vertex to the opposite side, **perpendicular** to that side. I'll label that altitude ***h***, for height. This creates two right triangles inside the original triangle. See below:



We should know by now that $\text{sine} = \dfrac{\text{opposite}}{\text{hypotenuse}}$. So we can say (truthfully) that:

$\sin A = \dfrac{h}{b}$ and that $\sin B = \dfrac{h}{a}$

Then a little algebra (or a diagonal slide) tells us that

$b \sin A = h$ and that $a \sin B = h$

Look! $b \sin A$ and $a \sin B$ both equal h. They must equal each other!

$b \sin A = a \sin B$

This result can be rearranged in several ways. My favorites are:

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ and $\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

Proof of the Law of Sines

Equation #1: $b\sin A = a\sin B$

Hey! Rearrange that, and it's the Law of Sines!

$\frac{\sin A}{a} = \frac{\sin B}{b}$ and $\frac{b}{\sin B} = \frac{a}{\sin A}$

Who would have imagined that? In any (and every) triangle, the **ratio** of the sine of an angle to the length of the opposite side is ***the same*** for any pair of angles and opposite sides. (Or, the ratio of the length of a side to the sine of the opposite angle … etc.)

Now, you could rearrange Equation #1 in other ways. Go ahead. Just don't break your algebra doing it.

In order to USE the Law of Sines to solve problems involving triangle parts (sides and angles), you need to **have** *three* parts, and then use those three parts to find the fourth piece.

Which parts you have is important. You need to have one pair where you know **both** the measure of an angle *and* the length of its opposite side, plus one other piece (a side or an angle) to find its partner (opposite angle or side) .

The Law of Sines can NOT help if you know the lengths of all three sides (SSS), but none of the angles; nor can the Law of Sines help if you know the measures of two sides and the measure of the angle that connects them (SAS). Fortunately, the Law of Cosines can solve those situations.